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that in the general case the form of these equations is not altered, and that the number of the constants remains the same. But if it be supposed that the internal moments are represented by the variation of a single function V, the present supposition differs from the hypothesis of independent action, in the absence of any restriction upon the form of V, whose coefficients are now perfectly arbitrary, and may, therefore, be assumed to satisfy at pleasure any given relations.

The Rev. Samuel Haughton communicated the following Abstract of a new Method of deducing Fresnel's Laws of Wave Propagation from a mechanical Theory.

In a memoir on a classification of elastic media, presented to this Academy in January, 1849, I deduced the general equations of motion resulting from the hypothesis, that the function on which they depend is a function of the nine differential coefficients of the displacements of each molecule. In that memoir I have also shown the possibility of the laws of wave propagation being the same in media of different molecular constitutions, and have given some examples in the theories of Light. The general function V used in that paper contains *forty-five* coefficients, and may be represented thus:

$$2V = \Sigma(\alpha_1^2) + 2\Sigma(\alpha_1\beta_1) + 2\Sigma(\alpha_2\alpha_3) + 2\Sigma(\alpha_1\beta_2), \qquad (1)$$

adopting the notation used in the memoir. The last term of this equation consists of *eighteen* terms, while each of the others contains *nine*. Among other hypotheses made by me at the time of writing the memoir, I assumed the coefficients of this last term to be equal in pairs, so as to reduce the total number of coefficients to thirty-six. The consequences which I deduced from this hypothesis were interesting, but I did not publish them in my memoir, as I could give no satisfactory reason for the hypothesis itself. As I conceived it at the time, it was only a mathematical assumption made to simplify my equations. Some days since, my friend Professor Jellett communicated to me a result at which he has lately arrived, and on referring to my note book I found that Mr. Jellett's theorem gave a *physical* reason for the hypothesis I have alluded to. So far as Mr. Jellett's investigation relates to this subject, it may be thus stated :

" 'If in a system of molecules the forces developed by the displacement of any two molecules be functions of *their relative* displacements only, and tend to restore them to their original positions ; the function V for such a system will contain thirty-six coefficients, the coefficients of $\Sigma(\alpha_1\beta_2)$ being equal in pairs.'

" This theorem evidently supplies the link which was wanting in my equations, which, perhaps, may not now be deemed unworthy of notice, as they may be shown to rest on a definite physical hypothesis.

" The note from which the following abstract is taken is dated December 26, 1848. I have slightly altered the notation, and prefixed two theorems which facilitate the understanding of what follows.

Theorem I.

" Let $(u_1, u_2, \&c.)$ be functions of (x, y, z), defined by the following equations:

$$\begin{array}{lll}
u_1 = \beta_2\gamma_3 - \beta_3\gamma_2 & v_1 = \gamma_2\alpha_3 - \gamma_3\alpha_2 & w_1 = \alpha_2\beta_3 - \alpha_3\beta_2 \\
u_2 = \beta_3\gamma_1 - \beta_1\gamma_3 & v_2 = \gamma_3\alpha_1 - \gamma_1\alpha_3 & w_2 = \alpha_3\beta_1 - \alpha_1\beta_3 \\
u_3 = \beta_1\gamma_2 - \beta_2\gamma_1 & v_3 = \gamma_1\alpha_2 - \gamma_2\alpha_1 & w_3 = \alpha_1\beta_2 - \alpha_2\beta_1
\end{array}$$

$(\alpha_1, \alpha_2, \beta_1, \&c.)$ denoting $\left(\frac{d\xi}{dx}, \frac{d\xi}{dy}, \frac{d\eta}{dx}, \&c.\right)$

" If the co-ordinates be changed into x', y', z', by changing the direction, without changing the origin, then the functions $(u_1, v_2, w_3, v_3 + w_2, w_1 + u_3, u_2 + v_1)$ will reproduce themselves by means of the following equations:

$$\left.\begin{aligned}
u_1 &= a^2u'_1 + b^2v'_2 + c^2w'_3 + bc\,(v'_3 + w'_2) + ac\,(w'_1 + u'_3) \\
&\qquad + ab\,(u'_2 + v'_1) \\
v_2 &= a'^2u'_1 + b'^2v'_2 + c'^2w'_3 + b'c'\,(v'_3 + w'_2) + a'c'\,(w'_1 + u'_3) \\
&\qquad + a'b'\,(u'_2 + v'_1) \\
w_3 &= a''^2u'_1 + b''^2v'_2 + c''^2w'_3 + b''c''\,(v'_3 + w'_2) \\
&\qquad + a''c''\,(w'_1 + u'_3) + a''b''\,(u'_2 + v'_1) \\
v_3 + w_2 &= 2a'a''u'_1 + 2b'b''v'_2 + 2c'c''w'_3 + (b'c'' + b''c')\,(v'_3 + w'_2) \\
&\qquad + (c'a'' + c''a')\,(w'_1 + u'_3) + (a'b'' + a''b')\,(u'_2 + v'_1) \\
w_1 + u_3 &= 2aa''u'_1 + 2bb''v'_2 + 2cc''w'_3 + (b''c + bc'')\,(v'_3 + w'_2) + \\
&\qquad (c''a + ca'')\,(w'_1 + u'_3) + (a''b + ab'')\,(u'_2 + v'_1) \\
u_2 + v_1 &= 2aa'u'_1 + 2bb'v'_2 + 2cc'w'_3 + (bc' + b'c)\,(v'_3 + w'_2) + \\
&\qquad (ca' + c'a)\,(w'_1 + u'_3) + (ab' + a'b)\,(u'_2 + v'_1).
\end{aligned}\right\}\quad(2)$$

These equations of transformation are identical with those for the transformation of x^2, y^2, z^2, $2yz$, $2xz$, $2xy$.

Theorem II.

"Let λ, μ, ν, ϕ, χ, ψ be functions defined by the following equations :

$$\begin{aligned}
\lambda &= \alpha_1^2 + \beta_1^2 + \gamma_1^2 & \qquad \phi &= \alpha_2\alpha_3 + \beta_2\beta_3 + \gamma_2\gamma_3 \\
\mu &= \alpha_2^2 + \beta_2^2 + \gamma_2^2 & \qquad \chi &= \alpha_3\alpha_1 + \beta_3\beta_1 + \gamma_3\gamma_1 \\
\nu &= \alpha_3^2 + \beta_3^2 + \gamma_3^2 & \qquad \psi &= \alpha_1\alpha_2 + \beta_1\beta_2 + \gamma_1\gamma_2
\end{aligned}$$

These functions will be changed, by a change of co-ordinates, into the following linear functions of similar quantities :

$$\left.\begin{aligned}
\lambda &= a^2\lambda' + b^2\mu' + c^2\nu' + 2bc\,\phi' + 2ca\chi' + 2ab\psi' \\
\lambda &= a'^2\lambda' + b'^2\mu' + c'^2\nu' + 2b'c'\phi' + 2c'a'\chi' = 2a'b'\psi' \\
\nu &= a''^2\lambda' + b''^2\mu' + c''^2\nu' + 2b''c''\phi' + 2c''a''\chi' + 2a''b''\psi' \\
\phi &= a'a''\lambda' + b'b''\mu' + c'c''\nu' + (b'c'' + b''c')\,\phi' + (c'a'' + c''a')\,\chi' \\
&\qquad + (a'b'' + a''b')\,\psi' \\
\chi &= aa''\lambda' + bb''\mu' + cc''\nu' + (b''c + bc'')\,\phi' + (c''a + ca'')\,\chi' \\
&\qquad + (a''b + ab'')\,\psi' \\
\psi &= aa'\lambda' + bb'\mu' + cc'\nu' + (bc' + b'c)\,\phi' + (ca' + c'a)\,\chi' \\
&\qquad + (ab' + a'b)\,\psi'
\end{aligned}\right\}\quad(3)$$

These equations are the same as the equations for transforming x^2, y^2, z^2, yz, xz, xy.

" Introducing into the equations derived from the function V (which contains thirty-six coefficients as above specified), the conditions that the vibrations shall be normal and transversal, I obtain the following relations among the constants:

$$\left.\begin{aligned}
&(a_1b_1) = 0,\ (a_1c_1) = 0,\ (b_2c_2) = 0,\\
&(a_2b_2) = 0,\ (a_3c_3) = 0,\ (b_3c_3) = 0\,;\\
&(b_1c_1) + (2a_1c_2) = (b_1c_1) + (2a_1b_3) = 0,\\
&(c_2a_2) + (2b_2a_3) = (c_2a_2) + (2b_2c_1) = 0,\\
&(a_3b_3) + (2c_3b_1) = (a_3b_3) + (2c_3a_2) = 0\,;\\
&(a_1a_2) = (b_1b_2) = (c_1c_2) - (a_3b_3),\\
&(b_2b_3) = (c_2c_3) = (a_2a_3) - (b_1c_1),\\
&(c_3c_1) = (a_3a_1) = (b_3b_1) - (a_2c_2)\,;\\
&(a_1^2) = (b_1^2) + (2a_1b_2) = (c_1^2) + (2a_1c_3),\\
&(b_2^2) = (c_2^2) + (2b_2c_3) = (a_2^2) + (2b_2a_1),\\
&(c_3^2) = (a_3^2) + (2c_3a_1) = (b_3^2) + (2c_3b_2).
\end{aligned}\right\}\ (4)$$

The notation will be understood by reference to my former memoir.

" These equations, twenty-four in number, reduce the function V to the following (in which X, Y, Z denote $\beta_3 - \gamma_2$, $\gamma_1 - \alpha_3$, $\alpha_2 - \beta_1$):

$$\left.\begin{aligned}
2V = {} & A\lambda + B\mu + C\nu + 2F\phi + 2G\chi + 2H\psi\\
& + P(X^2 - u_1) + Q(Y^2 - v_2) + R(Z^2 - w_3)\\
& + L\{2YZ - (v_3 + w_2)\} + M\{2XZ - (w_1 + u_3)\}\\
& \qquad + N\{2XY - (u_2 + v_1)\}
\end{aligned}\right\}\ (5)$$

" This function, containing twelve coefficients, may be reduced to a function of nine coefficients in two different ways, by the aid of the two theorems above given. In fact, let the two ellipsoids represented by the following equations be constructed:

$$\left.\begin{aligned}
&Ax^2 + By^2 + Cz^2 + 2Fyz + Gxz + 2Hxy = 1\,;\\
&Px^2 + Qy^2 + Rz^2 + 2Lyz + 2Mxz + 2Nxy = 1\,:
\end{aligned}\right\}(6)$$

it may be shown that the function V is so related to the two sets of axes of these ellipsoids, that by assuming either set of axes for axes of coordinates, we may destroy three coefficients of the function.

" Assuming as co-ordinate axes, the axes of the second of ellipsoids (6), the equation of the surface of *wave-slowness* is found to be the following :

$$(E-1)\left\{\begin{array}{c}\left[(x^2+y^2+z^2)\,(QRx^2+PRy^2+PQz^2)\right.\\ +(E-1)\{(Q+R)x^2+(P+R)y^2+(P+Q)z^2\}\\ \left.+(E-1)^2\right]=0\,;\end{array}\right\} \quad (7)$$

where

$$E = Ax^2 + By^2 + Cz^2 + 2Fyz + 2Gxz + 2Hxy.$$

" The equation is thus seen to be composed of two factors; the first, of the fourth degree, representing a surface with two sheets, which belong to the two transverse vibrations; the second, of the second degree, representing an ellipsoid which belongs to the normal vibration. The surface of the fourth degree may be shown to have sixteen multiple points, and consequently, its reciprocal polar, or wave surface, will be reduced from the thirty-sixth to the fourth degree.

" In the particular case in which the normal vibration vanishes, we shall have $E = 0$, and the surface of transverse vibrations will become Fresnel's wave surface, as is evident on inspection of equation (7). The function corresponding to this reduction will be

$$2V = P(X^2 - u_1) + Q(Y^2 - v_2) + R(Z^2 - w_3). \quad (8)$$

" In the last section of the memoir published in vol. xxii. Part I. of the Transactions of the Royal Irish Academy, I have directed attention to the fact, that the deduction of the laws of wave propagation is no proof of the truth of any mechanical theory of light, as this deduction may be made from several theories. There are, in fact, no less than *five* distinct theories already before the scientific public, which fulfil this essential condition, and I now publish a *sixth* mechanical theory, not because I think it superior to its predecessors, but in order, if possible, to direct attention to the unscientific state in which the question rests. It may be useful, with this view, to mention the various theories, which I shall do in chronological order : — First, Fresnel's theory ; second, M. Cauchy's theory, deduced from the mathematical equations of motion of a system of attracting and repelling molecules ; third and fourth, two theories of Mr. George Green, published in 1839 ; fifth, the late Professor Mac Cullagh's theory, published in the same year. To these may be added the theory now published. The existence of *five* rival theories is a formidable objection to each of the six, and until this objection is removed, none can claim to be the theory of Light.

"The *experimenta crucis* must be sought for in the laws of reflexion and refraction, as I have shown in my former paper. I am at present engaged in the investigation of the laws of reflexion, with the view of testing by experiment, if possible, the six theories of light. It may be interesting to observe, with respect to the direction of vibration, that in M. Cauchy's theory the vibrations are neither normal nor transversal, that in Fresnel's and Mr. Green's second theory, the vibrations are perpendicular to the plane of polarization, and that in Mr. Green's first theory, Professor Mac Cullagh's, and my own, the vibrations are parallel to the plane of polarization."

The Rev. Samuel Haughton communicated also the following note on the function peculiar to a system of attracting and repelling molecules.

"In my memoir on the equilibrium and motion of solid and fluid bodies, I have deduced the function from the supposition that the natural state of the body is one of free equilibrium.